Allocation of Seats and Voting Power in the Norwegian Parliament

By Jon Kåsa

Abstract:

In recent years there seems to be a trend in Norwegian politics that larger parties are getting bigger while smaller parties are getting less support. Could there be some mathematical explanation behind this trend? Given the apportionment method used in the current system in Norway, we investigated if indeed there would be advantageous for parties, in particular smaller ones, to merge together to increase combined voting power rather than staying separated, and if so how often is it advantageous. The results of our simulations do indicate that it is beneficial for smaller parties that on average is close to the lower threshold for obtaining additional "leveling seats" to merge together either with other smaller parties or medium sized parties.

The Apportionment Problem:

The idea of Democracy is interchangeable with equal or "fair" representation. This idea goes hand in hand with the ideal of one man, one vote. In a direct democracy or in a national referendum this ideal of one man, one vote is met. The problem arises in a representative democracy. What does equal representation look like in a political system with multiple parties, millions of votes and only a limited integer number of seats to be allocated? Throughout history, nations that have or are practicing a representative democracy have used a variety of methods and different models in the attempt to meet such ideals. There have of course been a lot of modifications and adjustments to whom in society that should have representation. Essentially, all the models of a representative democracy rely on either a federal system or a proportional representation system, or a combination of the two of them. In a federal system, states or provinces receives seats according to its population; the unit of representation is regional. The United States is using a federal system. While in a proportional representation system the unit of representation is political and parties win seats according to their votes. Israel is using such a system. CITE. Many countries in Europe such as Germany, Switzerland and Norway on the other hand is using a combination of the two where the unit of representation is bound both to territory and to party. In accordance to the ideal of one man, one vote, a state or a party should receive a number of representatives in proportion to its population or in proportion to the number of votes. The difficulty is what to do with the fractions. Consider a state with 8 seats to be allocated and three parties running in the election with the votes given by Table 1. The table also shows how many seats each party deserves by the use of proportionality. How should the seats be allocated? What method of allocation would be the most fair? One such method is the Webster method or a modified Webster method as used in Norway. The table below shows how the seats would be allocated by the Webster method on the three parties.

Table 1.

Parties	Number of Votes in Thousand	% of Total Votes	Deserved Number of Se
Party A	90	56%	4.48
Party B	45	28%	2.24
Party C	25	16%	1.28
Total	160	100%	8

Method for Allocation of Seats:

The Webster method for allocating seats is a divisor method of highest quotients that are widely used around the World. Among the countries are Bosnia and Herzegovina, Iraq, Kosovo, Latvia, New Zealand, Norway and Sweden. It has also been one of many methods that have been used in the U.S. The way the method works is after all the votes have been tallied the formula quotient = $\frac{V_i}{2 S_i + 1}$ calculates successive quotients. V_i stands for the total number of votes for party i, and S_i is the number of seats the party have been allocated so far. Whichever party has the highest quotient are allocated the next seat. The quotient for that party is then recalculated given their new seat total. The process is repeated until all seats have been allocated. The Webster method is unbiased between small and large parties. Some countries, Norway among them are also using a modified Webster method. It is done by changing the formula for calculating quotients for parties that have yet to be allocated any seats ($s = 0$) from V_i to $\frac{V_i}{1.4}$. It is a way to give slight benefits to larger parties and make it harder for smaller and potentially more extremist parties to gain power. Below is an illustration of the modified Webster method applied to a county with two running parties and three seats to be allocated.. The bold numbers are the quotients that resulted in a seat.

Table 2.

Parties	Number of Votes in Thousand	% of Total Votes	Deserved Number of S
Party A	90	56%	4.48
Party B	45	28%	2.24
Party C	25	16%	1.28
Total	160	100%	8

The Norwegian Voting System:

The Norwegian Parliament consists of 169 representatives, of them 150 are selected ~~from~~ directly from the 19 counties or constituencies while the last 19 are allocated according to the national distribution of votes. I will refer to these 19 seats as “leveling seats”. First, the way each county are assigned their respective number of seats in the parliament is by considering both the population and the area of each county. Each person in the state count for 1 point, while each km^2 counts for 1.8 points. After adding up for the number of people and area for each state, a Webster method of allocation is applied to allocate the 169 seats between the 19 counties. Now, 150 of these seats are allocated directly within each constituency by the use of the modified Webster method to determine how many of the seats within each county that is assigned to each party. One ambiguity of the Webster method to allocate seats is what do when the highest quotient is the same. In Norway, the constitution states that the party with the highest total vote count in the county of all the parties with the same largest quotient should be allocated the seat. If there are more than one party with the same highest quotient and total vote count in the county, the party that is going to be assigned the seat are chosen randomly among these parties. In the counties there are no lower threshold or any restriction on national basis for parties to be assigned seats within county where a party is popular enough to be allocated a seat by the Webster method. While for a party to get any of the 19 "leveling seats" the party need to have at least 4% of the total votes nationally. For all such parties, a new modified Webster allocation is performed by considering all the seats that is assigned to the parties that is greater than 4% and the parties total national votes. In other words, in the current parliament, there is only one party that has representatives and is below the 4% lower threshold. Thus the national allocation considers there to be 168 seats to be allocated between the seven remaining parties. The difference in the seats allocated by this national allocation and the total number of seats each of these parties obtained in the counties are the number of "leveling seats" a party get. In the case of parties that does exceptionally well in the counties, that is getting more total seats in the counties than considering the national allocation are not eligible for any "leveling seats". These parties will be taken out and their total seats in the states will be removed from the number of seats to be allocated in the national allocation and there will be performed a new allocation to determine the "leveling seats". These "leveling seats" is an attempt to give some more seats to parties that have been close to getting seats in several states.

Objective and Motivation:

In the sections above I have pointed out the difficulties that arises in representative democracies, the method of allocation of seats that is used in Norway (the Webster and modified Webster method) and how the Norwegian voting process works for the parliament. Still I have yet to mention too much about what part of the Norwegian system I am looking at. I am a Norwegian citizen and in recent years I have got the sense that smaller parties get smaller while the larger parties get larger. It make me question if the system is made in such a way that parties should merge together and in that way increase their combined voting power. Potentially, would it mean that Norway is going more towards a two party system like the U.S or the Great Britain? I wanted to investigate if this trend, that might be intuitively rational, is mathematically sound and what if it is advantageous for parties to merge, to what degree or how often does it increase voting power?

Shapley-Shubik Voting Power:

What does it mean to be advantageous for parties to merge to together? One way would be to consider that the parties obtain more seats by coming together than their combined seats when their apart, but often in politics we are interested in power or voting power. How can we measure a party's voting power? In mathematics there are developed several methods to determine power, one of the most commonly used are the Shapley-Shubik Power Index. It consider a party to have power when the party's vote is pivotal to pass a resolution. In other words whenever a party can turn a losing resolution into a passing resolution, it possess actual power. The Shapley-Shubik Power Index is then the probability for a party to be pivotal or decisive in a decision. The Shapley value is determined by considering all orderings of the parties and in each ordering determine what party is turning the coalition of parties before from a losing coalition into a winning coalition. Then find the probability for each party to be pivotal and that is the Shapley value or the voting power for a party. Table 3 and 4 below illustrates how the Shapley values are calculated for an example of a parliament consisting of three parties where party A, B and C has 5, 2, 1 seats respectively and the quota to pass a resolution is 6 votes (seats). Notice that even though party B has twice as many seats as party C, they have the same voting power. Thus an increased number of seats allocated don't directly correspond to an increase in voting power. Sometimes it can even lead to the opposite.

Table 3.

Ordering	Pivotal
A B C	B
A C B	C
B A C	A
B C A	A
C A B	A
C B A	A

Table 4.

Party	Number of times Pivotal	Shapley Shu Index
A	4	4/6 = 2/3
B	1	1/6
C	1	1/6

The Simulation:

In order to evaluate if and how often it would be advantageous for parties to merge together to increase combined voting power, we needed to look beyond only the historical data since the current political system was implemented in 1952. We decided to create an algorithm in order to look at simulations of 10000 elections. To create a realistic model, we had to base it on real data. In politics especially it does seem to be a trend that previous elections affect what a person will vote next. On SSB webpage, the webpage of the central statistics bureau of Norway, could provide the data necessary. In appendix B the file of the data spreadsheet acquired from their webpage is available. It contain all the information for the 7 largest parties in Norway. The list provides each party's vote count within each county for each election since 1953. I decided to look at only these seven parties because they were the only parties that have had more than one representative at least one time within these past elections. Then we used the DistributionTest function we coded to determine how well the data for each party within each county fitted a normal distribution according to an Anderson Darling test. By experimenting with as large sample of data as possible, the optimal choice was to select all the data from the elections since 1980. A larger sample from previous year yielded a lower percentage of data for parties within a particular county that fitted a normal distribution. With the choice of data from 1980 around 82% of all the sample data a normal distribution could not be rejected as a fit. Therefore I made the assumption that for the simulation all the sample data would be selected from a normal distribution given the historical data from 1980. Thus my simulation for each simulated election selects a vote count for each party within each state based upon a normal distribution given by the historical mean and standard deviation from the election data since 1980. For each simulated election the algorithm would compare the seats allocated and the voting power for the parties of interest by coming together versus staying apart. In the process of computing voting power I used the floor of the number of total representatives in the parliament divided by two as on all resolutions beside changes in the constitution only a simple majority is needed.

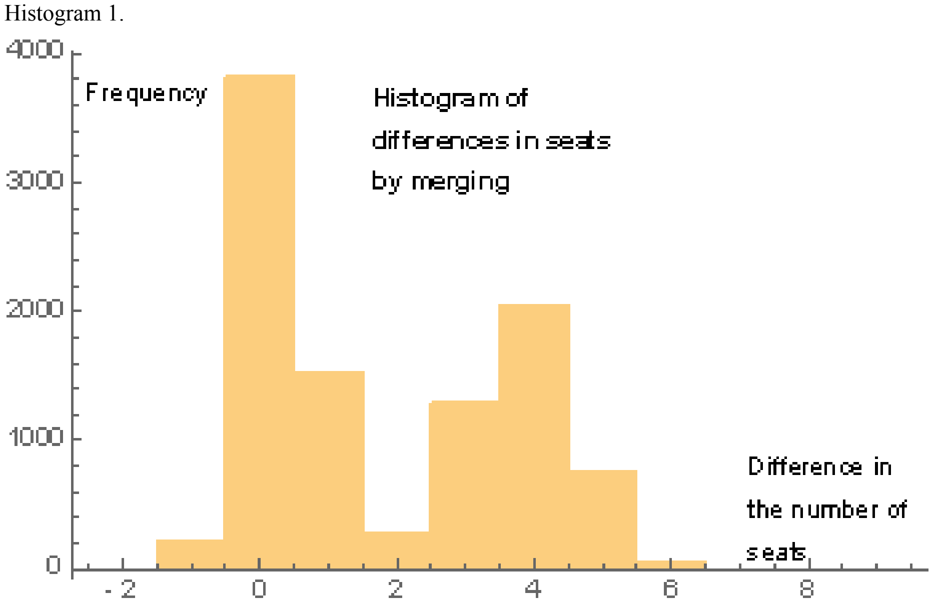
The Political Landscape in Norway and Parties of Interest:

Among the seven selected parties in the simulation here is a list of the parties from more liberal to more conservative; SV, AP, SP, Krf, V, H, Frp. To answer the question if and how often it is advantageous for parties to merge together we thought there were three scenarios of importance. These three scenarios are centered on what we believe to play a decisive role in the number of seats allocated and voting power, namely the “leveling seats”. The three scenarios arises out of where the parties are in relation to the lower and upper threshold for these additional seats. The first scenario would then be that at least one of the parties are historically sometimes above and sometimes below the threshold of a national vote count of 4%. Therefore the first merging of interest was to look at if V and Krf came together. Both of these two parties are more towards the conservative side and are closely related in their political views and both of the parties are sometimes above or below the threshold. Currently they often vote together as a block on many issues and present one of the more realistic set of parties to merge. It is also one of the cases that is the most related our motivation for this study. Along the same lines we decided to look at merging SV and SP as well. They are both liberal parties and a realistic option, but in comparison to above only SV is close to the threshold. The second scenario would be to look at two parties that is always above the threshold, but even when they come together they won’t be big enough to often hit the upper threshold. One such merging were to consider merging H and SP. That these two parties would actually merge is quite unlikely as they represent two opposing political views, but the intent was that it could tell us something about middle sized parties merging together. The third scenario are then to look at two parties that are big enough that when they merge, they might occasionally pass the upper threshold. The only such option would be to consider that AP and H would merge, the two largest parties in Norway. Once again this would be a very unlikely event, but could give us some information on the nature of the system.

Data and Analysis:

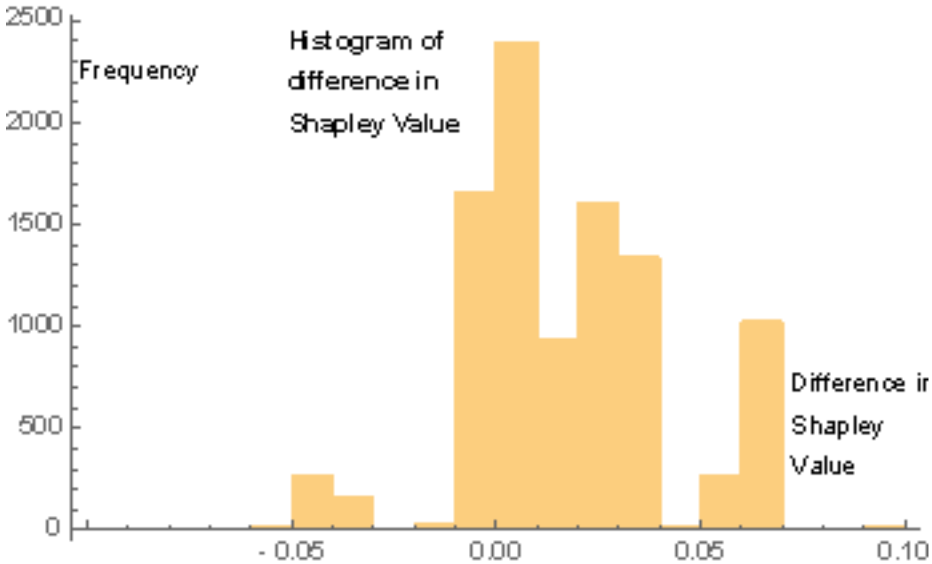
Here are the results for merging Krf and V when comparing the number of seats that are allocated. Without even considering to what degree it is beneficial, as one can observe from the histogram at worst the parties will lose one seat while they can gain up to six seats. The histogram shows that in only 2% of the times do Krf and V loose seats when coming together while they gain seats in 60% of the times.

Histogram 1.



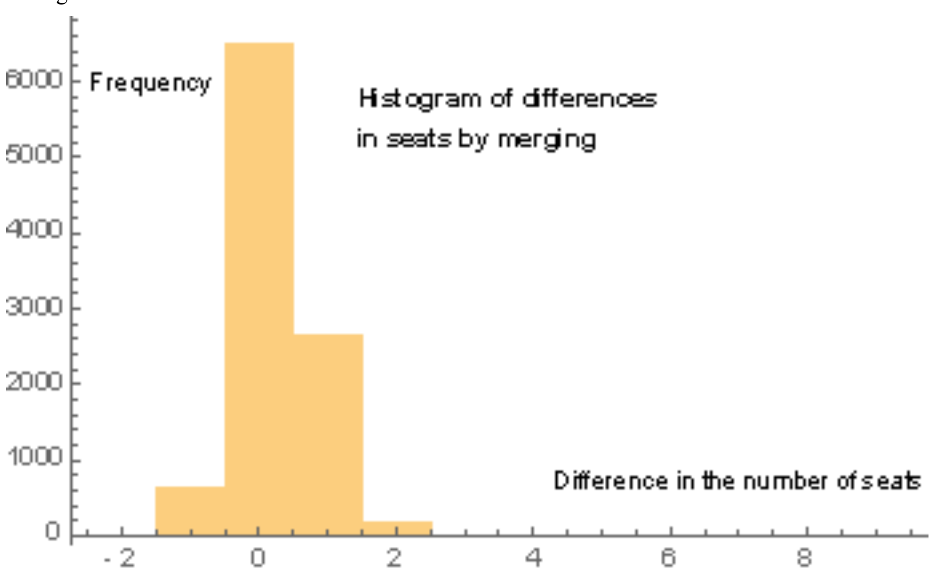
Here are the results for merging Krf and V when comparing the voting power they gain by coming together. The histogram shows that even though only 2% of the times do Krf and V loose seats when coming together they lose power 22% of the times. They do gain power about the same percentage as they gain seat with 59% of the times.

Histogram 2.



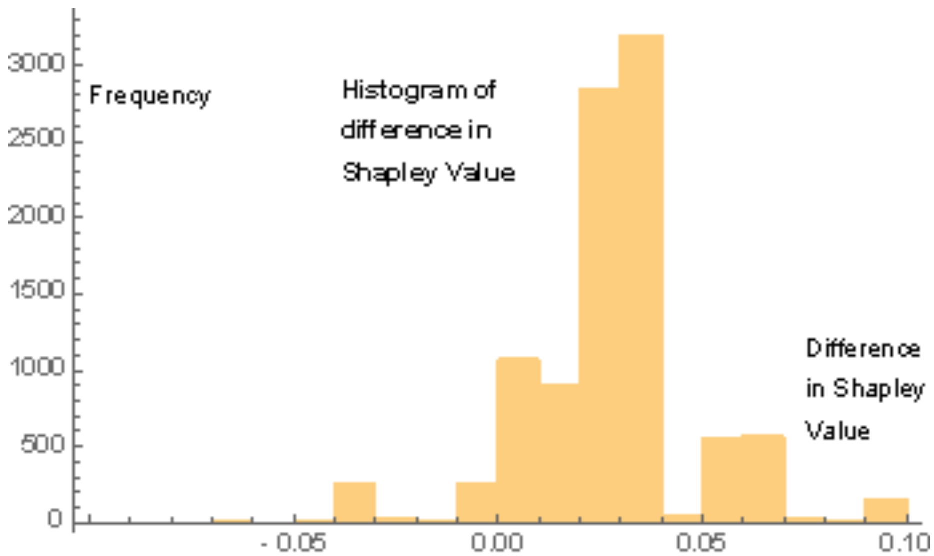
Similar results for when SP and SV merge together. Here are the results for merging SP and SV when comparing the number of seats that they are allocated coming together. Though there are now at most two seats to gain, they only lose seats in 6% of the cases while thei gain seats 29% of the times.

Histogram 3.



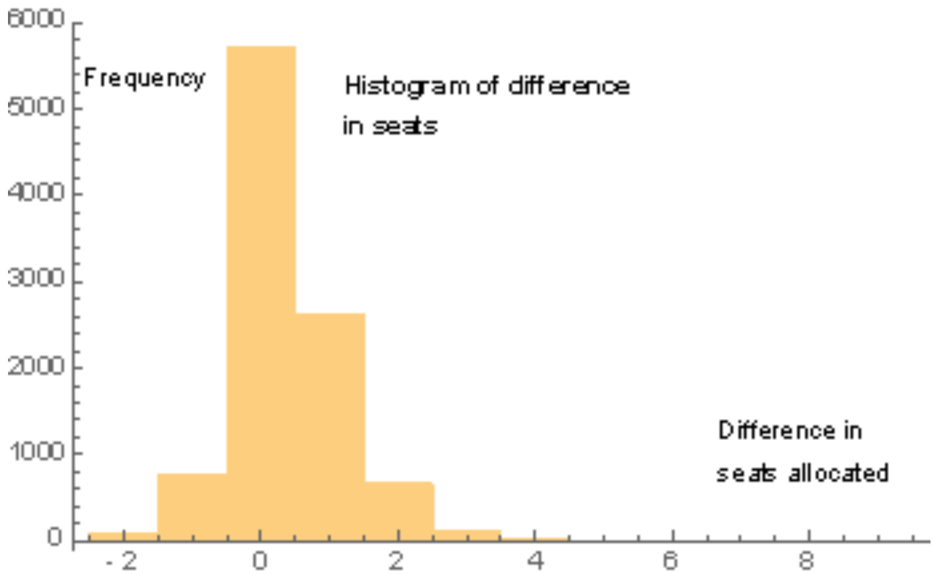
Here are the results for merging SP and SV when comparing the voting power they gain by coming together. The histogram shows similarly they loose voting power in 6% of the times they gain voting power in 88% of the times! It is a similar trend as Krf and V coming together in the way it is in both cases more benefitial than not both in seats allocated and power to come together.

Histogram 4.



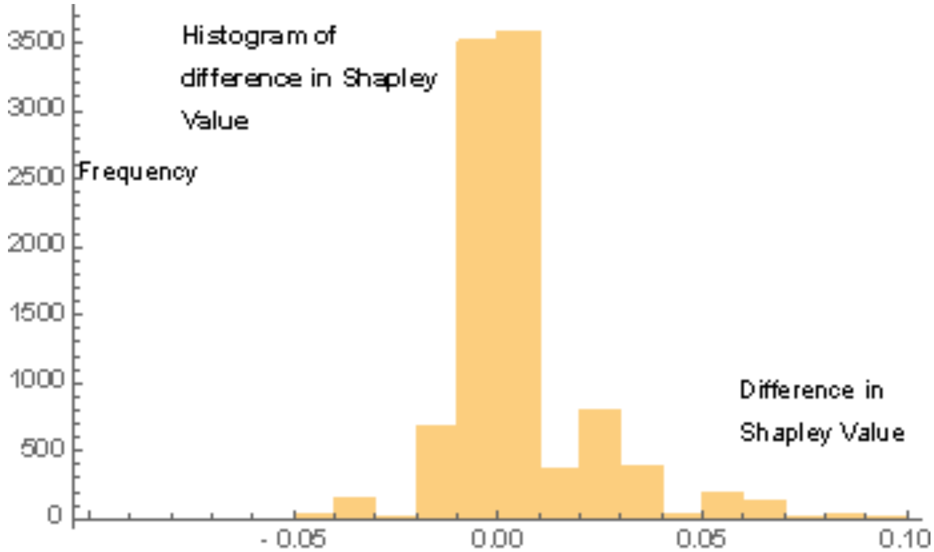
Here are the results for merging H and SP. The histogram shows that in 9% of the times they lose seats while they gain seats 34% of the times.

Histogram 5.



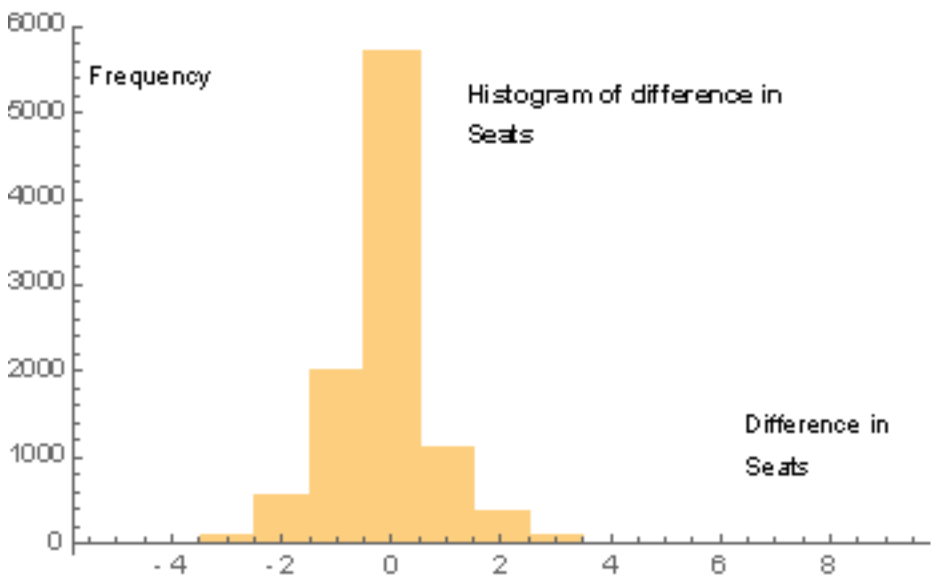
While when we compare to the difference in voting power by merging H and SP we see an oppsoite trend that earlier. Considering only the seats allocated it appears to be beneficial to come together, but when we look at voting power more often it is a disadvantage than the oppsoite with a loss of power 44% of the times and only a gain of power 26% of the times.

Histogram 6.



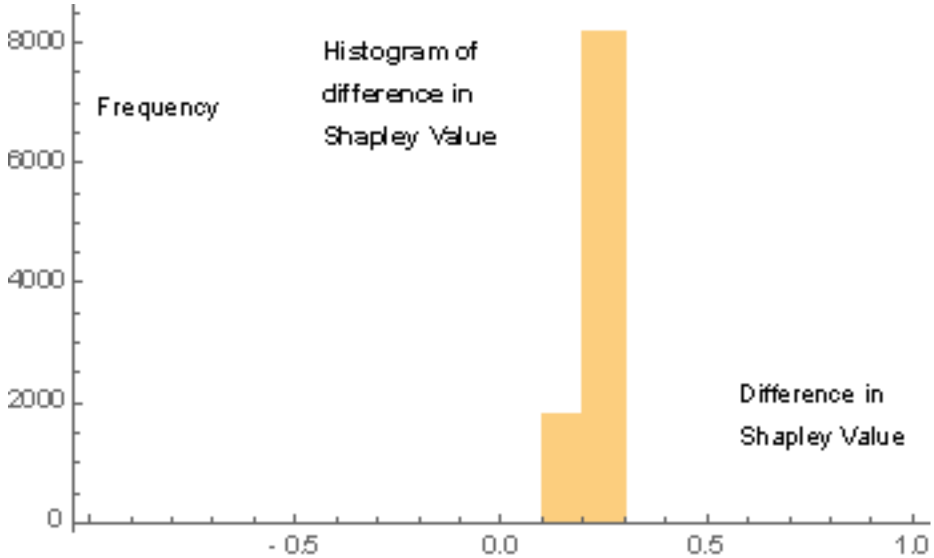
Here are the results for the difference in seats allocated for merging AP and H. The histogram shows that more often it is a disadvantage than advantage in the number of seats allocated with 26% versus 16%.

Histogram 7.



The results in difference in voting power on the other hand is quite astonishing for merging AP and H. Though in most cases when it comes to number of seats it is either no change in seats or more often negative considering seats to come together, the histogram below shows that it is always beneficial to come together if one are to consider voting power.

Histogram 8.



The results of the simulations above project our intuition that for parties that sometimes are below the threshold to get ”leveling” seats on their own increase their both their number of seats and voting power by merging with other parties, to ensure that they always receive “leveling seats”. For example Krf and V or SP and SV as merging SP and SV yield similar results. For parties that are large enough to ensure that they receive “leveling” seats on their own, like DNA, SP, FRP, and H, there seem to be little or no incentive to merge together with other parties by looking at combined voting power even though looking at the number of seats there seem to be a trend that it could be beneficial. It does seem that big parties might benefit by merging with other big parties even though it is quite unlikely considering their opposing political views.

Sources:

Balinski, Michel L.; Peyton, Young (1982). *Fair Representation: Meeting the Ideal of One Man, One Vote*.

“Valg.” *Ssb.no*. Department of Finance, n.d. Web. 24 June 2016. <http://www.ssb.no/valg>.

Appendix A:
Code for the Simulations

Seats ordered after county:

seats = {8, 16, 18, 6, 6, 8, 6, 5, 3, 5, 13, 15, 3, 8, 9, 4, 8, 5, 4};

counties = {Østfold,Akershus,Oslo,Hedmark,Oppland,Buskerud,Vestfold,Telemark,AustAgder,VestAgder,Rogaland,Hordaland,SognogFjordane,MøreogRomsdal,SørTrøndelag,NordTrøndelag,Nordland,Troms,Finnmark};

The function for the allocation of seats according to method used by the Norwegian Government.

```
(* NOS is number of Seats to be assigned *)
(* V is the number of votes for each party *)
(* ST is a list of the number of seats given to the parties and thus ST[[i]] is
 the number of seats given to party i *)

AllocationOfSeats[NOS_, V_] := Module[{ST, s, v, L, RestV, l, lv, lp, lm, t},
  s = Length[V]; (*This is the number of parties*)
  v = V;
  ST = Table[0, {i, 1, s}];
  While[Total[ST] ≤ NOS - 1,
   L = Table[If[ST[[i]] == 0, v[[i]]/1.4, v[[i]]/(2 ST[[i]] + 1)], {i, 1, s}];
   (* L is quotients for each party given the number of seats according to the
    modified Webster divisor method and determines the priorities for the parties
    to receive the next seat *)
   (*Print[N[L]];*) (* Used for the print in the demonstration *)
   l = {};
   lp = Flatten[Position[L, Max[L]]];
   lm = RandomChoice[Select[lp, V[[#]] == Max[V[[lp]]] &]];
   ST[[lm]] = ST[[lm]] + 1
  ];
  ST]
```

Demonstration:

For the following demonstration I added an additional print command to illustrate how the list L looks like and that the code follows the Norwegian method to deal with equal quotients and if two parties have the same quotient and total vote count. The demonstration is considering the allocation of seats according to the method above on three parties in a state with 6 seats to be allocated. The number of votes for parties 1,2 and 3 is given in the format {Votes for Party 1, Votes for Party 2, Votes for Party 3} and in the first case it is {50,20,30}. This first demonstration illustrates that if there are two equal quotients, then the party with the highest vote count is assigned the seat.

```
AllocationOfSeats[6, {50, 20, 30}]

{35.7143, 14.2857, 21.4286}

{16.6667, 14.2857, 21.4286}

{16.6667, 14.2857, 10.}

{10., 14.2857, 10.}

{10., 6.66667, 10.}

{7.14286, 6.66667, 10.}

{3, 1, 2}
```

The second demonstration illustrate that there is up to chance what party is assigned the seat if there are more than one party with the heighest quotient and the same number of votes. Only modification from above is that party 3 now has 50 units of votes instead of 30.

```
AllocationOfSeats[6, {50, 20, 50}]
```

```
{35.7143, 14.2857, 35.7143}
{35.7143, 14.2857, 16.6667}
{16.6667, 14.2857, 16.6667}
{16.6667, 14.2857, 10.}
{10., 14.2857, 10.}
{10., 6.66667, 10.}
   {2, 1, 3}
```

```
AllocationOfSeats[6, {50, 20, 50}]
```

```
{35.7143, 14.2857, 35.7143}
{16.6667, 14.2857, 35.7143}
{16.6667, 14.2857, 16.6667}
{10., 14.2857, 16.6667}
{10., 14.2857, 10.}
{10., 6.66667, 10.}
   {3, 1, 2}
```

The next function is the apportionment function that you input the vote counts for each party within each state and it will return how many seats each party is allocated.

```
(* NOTS is number of total seats to be assigned *)
(* ListStates is a list of seats allocated to each state *)
(* ListPV is a list of lists with each party's vote count in each of the states,
same odering of the states as in ListStates *)
(* Merg is a list of the positions of the two parties you want to merge together,
currently the code is written only for two parties to merge together *)
(* My apportionment function is made in such a fashion that it checks for any
 input about what two parties you want to merge, and if no such command is given,
it will find the apportionment only for the original parties *)
```

```
Apportionment[NOTS_, ListStates_, ListPV_] :=
 Module[{quota, n, m, TotRestV, s, ST, Tot, TotPV, re, A, A1},
  n = Length[ListStates];
  m = Length[ListPV[[1]]];
  ST = Table[0, {i, 1, m}];
  Tot = Table[0, {i, 1, m}];
  Do[
   ST = ST + AllocationOfSeats[ListStates[[i]], ListPV[[i]]], {i, 1, n}];
  TotPV = Map[Total, ListPVᵀ];
  Do[If[TotPV[[i]] < 0.04 * Total[TotPV]
     , TotPV[[i]] = 0], {i, 1, m}];
  (*TotPV is the total party votes for all parties with more than 4%,
  the rest I set to 0 so the further seat allocation works out *)
  A = AllocationOfSeats[NOTS, TotPV];
  Do[If[A[[j]] < ST[[j]], TotPV[[j]] = 0], {j, 1, m}];
  (*The Norwegian System says that any party that gets more votes in the national
   allocation than what they got in the state wise one should not be entitled
   to any leveling seats, set the party total vote to 0 to adjust for it *)
  re = 0;
  Do[If[TotPV[[j]] == 0, re = re + ST[[j]]], {j, 1, m}];
  (*This is the seats that is allocated to the parties that cannot get any
   leveling seats and it needs to be subtracted from the national seats to be allocated *)
  A1 = AllocationOfSeats[NOTS - re, TotPV];
  Do[
   If[A1[[j]] == 0, ST[[j]] = ST[[j]],
    ST[[j]] = A1[[j]]], {j, 1, m}];
  ST]


(* NOTS is number of total seats to be assigned *)
(* ListStates is a list of seats allocated to each state *)
(* ListPV is a list of lists with each party's vote count in each of the states,
same odering of the states as in ListStates *)
(* Merg is a list of the positions of the two parties you want to merge together,
currently the code is written only for two parties to merge together *)
(* My apportionment function is made in such a fashion that it checks for any
 input about what two parties you want to merge, and if no such command is given,
it will find the apportionment only for the original parties *)

p = {Ap, SV, Sp, Krf, V, H, Frp, MDG};
(*The order of the parties for this use of the apportionment that comes from the
 distributions*)
```

```
ApportionmentComparison[NOTS_, ListStates_, ListPV_, Merg_] :=
 Module[{quota, n, m, TotRestV, s, ST, Tot, TotPV, re, A, A1, l, STM, TotPVM, net},
  n = Length[ListStates]; (* Number of States *)
  l = ListPV;
  If[Length[Merg] ≠ 0,
   m = Length[ListPV[[1]]] - Length[Merg] + 1;
   Do[
    AppendTo[l[[i]], Total[ListPV[[i]][[Merg]]]];
    (*For each state, add the combined vote counts for the parties you want to
     merge at the end, and delete the positions of the parties vote count for list l *)
    l[[i]] = Delete[l[[i]], {Merg}ᵀ]
    , {i, 1, n}];

   STM = Apportionment[NOTS, ListStates, l]];
  ST = Apportionment[NOTS, ListStates, ListPV];

  net = Last[STM] - Total[Table[ST[[Merg[[i]]]], {i, Length[Merg]}]];
  {net, {ST, STM}}]

(* Distribution is a function that determines the means and standard deviations
 for a normal distribution of the historical data of votes for each party in each state *)
(* NrYears is the number of years in my data collection,
in my case that was all elections since 1953 *)
(* Y is the number of terms, 4 years, since the first election that I decided to look at,
for my case I chose to start at the 7th elections since 1953 since thats when I
 had the most distributions not being dismissed as normal *)

Distribution[data_, NrParties_, NrYears_, NrStates_, Y_] := Module[{s, l, selecteddata},
  s = 0;
  l = {};

  Do[Do[
    selecteddata = data[[Y + s * NrYears ;; NrYears + s * NrYears]];
    x̄ = Mean[selecteddata];
    σ̂ = √(CentralMoment[selecteddata, 2]);
    AppendTo[l, NormalDistribution[x̄, σ̂]];
    s ++
    , {i, 1, NrParties}], {j, 1, NrStates}];
  N[l]
 ]

(* This is your function to find the Power Indices,
currently I have only modified it to look at Shapley values,
and an ordering detail so that I can bring in a list that is not in decending
 order and get the shapley values back *)
```

```
MWCValues[quota_, weights_] :=
 Module[{α, β, c, cp, γ, φ, n, ν, ts, wbar, wn, x, xi, yp, z, weight, order, sres},
  weight = Sort[weights, Greater]; (* Modifications *)
  order = Ordering[weights];
  n = Length[weight];
  z = 1; ν[1] = 1; wbar[1] = weight[[1]];
  Do[If[weight[[i]] == weight[[i - 1]], ν[z] ++, ν[++z] = 1;
    wbar[z] = weight[[i]]], {i, 2, n}];
  φ = β = γ = Table[0, {z}];
  Do[
   Do[c[w, t] = 0, {w, 0, quota - 1}, {t, 1, n - 1}]; c[0, 0] = 1; ts = 0;
   Do[α[t] = 0, {t, 0, n - 1}];
   Do[
    yp = ν[x] - If[x == xi, 1, 0];
    Do[
     If[c[w, t] > 0,
      cp = 1;
      Do[
       cp = cp (yp - y + 1) / y;
       wn = w + wbar[x] y;
       If[wn ≤ quota - 1,
        c[wn, t + y] += c[w, t] cp;
        α[t + y] = Max[α[t + y], wn]];
       If[quota - wbar[xi] ≤ wn ≤ quota - 1,
        φ[[xi]] += (t + y) ! (n - t - y - 1) ! c[w, t] cp;
        β[[xi]] += c[w, t] cp];
       If[quota - wbar[xi] ≤ wn ≤ quota - 1 + Min[0, wbar[x] - wbar[xi]],
        γ[[xi]] += (1 / (t + y + 1)) c[w, t] cp]
       , {y, 1, yp}]]
     , {t, ts, 0, -1}, {w, α[t], 0, -1}];
    ts += yp
    , {x, 1, z}];
   , {xi, 1, z}];
  Do[sh[wbar[i]] = φ[[i]]; bz[wbar[i]] = β[[i]]; dp[wbar[i]] = γ[[i]], {i, z}];
  sres = weight; (* Modifications *)
  sres[[order]] = Reverse[weight];
  {sres, Table[N[sh[sres[[i]]] / n!], {i, n}]}]
```

Demonstration of MWCValues independently of the other functions. Lets consider the

```
(* This is my simulation function that determines the normal distributions and
 create a random variate for the vote count for each party in each state,
then finds the allocations of seats and compare number of seats for the parties
 that is looked at for the merging,
and comparing the shapley values of the combined party versus the sum of the
 indivudual ones. It also returns the seat fraction versus the vote fraction for each party,
both for all seven parties as well as for the merging party as well as the shapley
 value versus the votefraction *)
```

```
OneSimulation[data_, NrParties_, NrYears_, NrStates_, NOTS_, ListSeats_, Y_, Merg_] :=
 Module[{l, pl, s, fl, rv, app, ShValues, net, votefraction, seatfraction, totvot,
   totvotmerge, votefractionmerge, quota, seatvsvote, powervsvote, ratiovsvote},
  quota = Ceiling[NOTS/2];
  l = Distribution[Flatten[data], NrParties, NrYears, NrStates, Y];
  pl = {};
  fl = {};
  s = 0;
  Do[Do[
    rv = RandomVariate[l[[i + s*NrParties]]];
    If[rv ≥ 0, AppendTo[pl, rv], AppendTo[pl, 0]], {i, 1, NrParties}];
   AppendTo[fl, pl];
   pl = {};
   s++
   , {j, 1, NrStates}];
  totvot = Total[fl];
  votefraction = totvot/Total[totvot];

  totvotmerge = Delete[AppendTo[totvot, Total[totvot[[Merg]]]], {Merg}^T];

  votefractionmerge = totvotmerge/Total[totvotmerge];
  app = ApportionmentComparison[NOTS, ListSeats, fl, Merg];
  seatfraction = app[[2]]/NOTS;
  ShValues = Table[Last[MWCValues[quota, app[[2]][[i]]]], {i, Length[app[[2]]]}];

  seatvsvote = Table[{votefraction[[i]], seatfraction[[1]][[i]]}, {i, Length[seatfraction[[1]]]}];
  powervsvote = Table[{votefraction[[i]], ShValues[[1]][[i]]}, {i, Length[ShValues[[1]]]}];
  net = Last[ShValues[[2]]] - Total[ShValues[[1]][[Merg]]];
  {app[[1]], net, seatvsvote, powervsvote, seatfraction[[1]]/votefraction, seatfraction[[2]]/votefractionmerge,
   ShValues[[1]]/votefraction, ShValues[[2]]/votefractionmerge}
 ]
```

Function to determine how many and how the distributions that doesn’t fit a normal distribution looks like.

```
DistributionTest[data_, NrParties_, NrYears_, NrStates_, y_] :=
 Module[{s, l, selecteddata, ADT, sd},
  s = 0;
  l = {};
  sd = {};
  Do[Do[
    selecteddata = data[[y + s * NrYears ;; NrYears + s * NrYears]];
    ADT = AndersonDarlingTest[selecteddata, NormalDistribution[μ, σ]];
    If[ADT < 0.05,
      AppendTo[l, {i, j, ADT}];
      AppendTo[sd, selecteddata]
      (*Print[{ADT,i,j}]*)
      (*Print[SmoothHistogram[selecteddata,1000]]*)]
     s ++
    , {i, 1, NrParties}], {j, 1, NrStates}];
  {l}
 ]
```

Appendix B: Data Collection

I didn’t know how to add an excel file here so I attached it to the email.